Development of SFEcon's Interest Rate Calculation

The Issue of Financial Control:

"SFEcon is an inquiry into the sources of order and stability in capitalist systems." Its causality is fundamentally neoclassical, albeit with an exception or two.

Prospects for demonstrating stability in numerical emulators of neoclassical causation have been emphatically disparaged by economics' mainstream – most significantly by the late Hugo Sonnenschein. The reasoning here usually follows from an observation that a capitalist system is composed of two positive feedback loops: one for material production; and another for finance.

1. Neoclassicism originates in the premise that utility functions are always increasing as asset usage increases, even while the rates of production's increase are always decreasing at the margin. As more assets are produced, still more assets can be produced. Thus more in the way of economic theory is required to explain why material goods should not be theoretically expected to proliferate without limit;

2. The 'more economic theory' generally posits the existence of an active financial control mechanism. But such control mechanisms are, in contrast to the weakly self-reinforcing processes of production, strongly self-reinforcing: unserviced debt positions, as well as unredeemed savings positions, grow exponentially at the rate of interest;

This logic is difficult to impeach – at least on the plane of reason. Positive feedback, being an notoriously ornery critter to control, it is difficult to imagine how any arrangement of two such systems might effectively tame them both. But that is what we are here to do.

Model 0:

SFEcon Model 0 is an elementary demonstration of financial control operating on a general, international input/output structure. It exhibits “the continuum of all chaotic physical and financial states, as well as disequilibrium prices, by which an economic system might efficiently guide itself into a new, previously unknown, unique, and equifinal Pareto optimum.”

These demonstrations limited financial intermediation to one interest rate and one investment term for each economy’s composite national investment portfolio. The intermediary himself was transparent in that he earned no profits with which to support the consumption of any assets. All profits earned by the industrial sectors are transmitted, in toto, to the household sectors in the form of passive income. The intermediary’s only function is to maintain contra-accounts against the investment positions of all the industrial sectors, and against the savings positions of all the household sectors.

Investment κ is the sum of all that industries have spent but not yet earned, and savings γ is the sum of all that households have earned but not yet spent. These sums include the history of all the interest payments from industries to households, hence there is no requirement that κ and γ be equal in magnitude – which constitutes our essential departure from neoclassicism.

Even more contentious is our practice of incorporating the magnitudes of κ and γ in a novel formulation of net present value NPV. Net present value’s most familiar expression is given thusly in SFEcon’s notation:

$$\text{NPV} = \frac{1}{(1-\iota)^{\text{T}}} \quad (1$$

It is composed by the investment term T and the interest rate $-\iota$. (Interest rates enter our system as negative quanta in order to signify the positive feedback aspect of financial intermediation.)

SFEcon's second treatment of net present value formulates NPV in terms of the intermediary's contra-accounts κ and γ:

$$\text{NPV} = \frac{\gamma/\text{T}}{\gamma/\text{T} + \iota\cdot\kappa} \quad (2$$

In this formulation, NPV is given as the ratio of a financial flow now, γ/T, that is given in exchange for the expectation of a financial flow, $\gamma/\text{T}+\iota\cdot\kappa$, in the future. (SFEcon's sign convention has κ computed as a negative and γ computed as a positive.)

Once the operative sense of our algebraic NPV is understood, this equation can be usefully simplified as follows:

$$\text{NPV} = \frac{1}{1 + \iota\cdot\text{T}\cdot\kappa/\gamma} \quad (3$$

Here we see net present value NPV has been re-interpreted as a function of the interest rate $-\iota$, investment term T, and an elementary interpretation of leverage, κ/γ.

The essential premise of SFEcon's formulation upon financial intermediation is that the first, transcendental expression of NPV must always equal the second, algebraic expression of NPV, which incorporates the intermediary's contra-accounts κ and γ.

Equations 1 and 3 comprise a system incorporating the three parameters to be continuously specified by financial intermediation, viz.: NPV, the interest rate $-\iota$, and the investment term T. This specification accesses the two state variables that Model 0's intermediary is always reformulating, viz.: investment κ and savings γ.

So SFEcon's portrait of financial intermediation begins with two equations and three unknowns. Our strategy for completing our earliest models involved some exterior specification of the interest rate $-\iota$. Primitive approaches to Model 0 demonstrated that treating $-\iota$ as a completely exogenous parameter produced stable emulations. The final version of Model 0 introduced the somewhat more satisfying strategy of defining the interest rate $-\iota$ as the current collective internal rate of return for all a model's industrial sectors.

With ι, κ, and γ given to the intermediary's calculations, we can readily eliminate NPV from equations 1 and 3 . . .

$$(1-\iota)^{T} = 1 + \iota \cdot T \cdot \kappa/\gamma \qquad (4$$

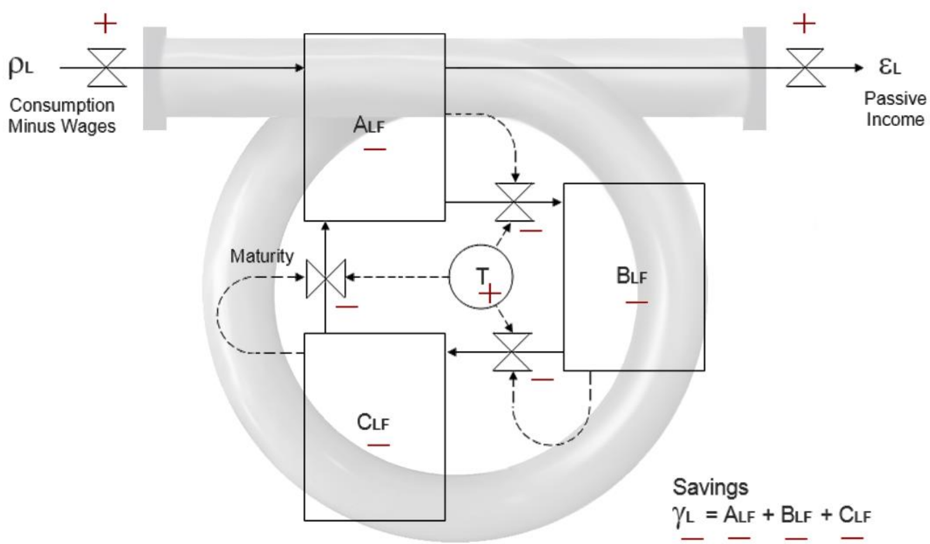
. . . thereby arriving at a single equation in one unknown, the investment term T.

We must, however, note that this system offers no possibility of solving Equation 4 by direct algebraic means. Transcendental, and more static algebraic formulations, constitute two mathematically estranged ways of looking at the same world; and their combinations do not always eventuate the most the most convenient mathematical systems.

Isolating the investment term τ in Equation 4 entails a mercifully straightforward application of the Newton-Raphson technique; and the recursive processes required by this technique are readily incorporated in SFEcon's Euler methodology for dynamic emulations.

The Circuit of Loanable Funds:

Our notion of the investment term τ can be reinforced through some consideration of the 'savings loop' represented below. Here we see a 'signal path' for that portion of a household sector's net cash flows that are saved.



A household sector's net cash flows ρ are shown entering a higher-ordered pipeline delay controlled by the variable τ, circumnavigating a mathematical loop, and emerging as passive income ε τ years later. SFEcon's sign convention has the household sectors' savings γ recorded as negative quanta, so as to be offset by the intermediary's positive savings contra-account.

Net cash flows ρ are computed as consumption minus wages, which are generally positive quanta: total consumption exceeds wages, with the difference being supplied by passive income. The system's positive driving function ρ operates to diminish the magnitude of the loop's negative internal state variables A, B, and C.

Passive income ε more or less restores savings as they are being reduced by $\rho =$ consumption-minus-wages. This positive rate of outflow ε operates to diminish the loop's negative internal state variables A, B, and C, thereby depicting an increase in γ's (negative) magnitude.

SFEcon's continuously varying investment term τ (disclosed by solution to Equation 4) controls passive income ε by speeding up or slowing down the rate at which funds flow through the savings loop.

SFEcon Model 0's financial intermediary, presumed to operate with transparency, requires no income with which to purchase assets. Hence he would pass along all of his harvest of profits $-\iota\kappa$ into passive income ε. Thus the $-\iota\kappa$ term in Equation 2's denominator can be replaced by ε, and this equation can then be rearranged so as to disclose passive income in terms of variables available to the intermediary:

$$\varepsilon = \left[\frac{1 - 1/\mathrm{NPV}}{\mathrm{T}}\right] \cdot \gamma \qquad 5)$$

An Interpretation of Financial Collapse:

This is an apt point at which to interject some commentary on SFEcon's interpretation of financial collapse. Though Equation 4 cannot be solved in mathematically closed-form, its antecedent Equations 1 and 3 can portray the essence of financial collapse when each is solved for the investment term τ.

$$\tau = \frac{-\ln(\text{NPV})}{\ln(1-\iota)} \qquad (6$$

$$\tau = \frac{\gamma}{(-\iota\kappa)}\left[\frac{\text{NPV}-1}{\text{NPV}}\right] \qquad (7$$

The transcendental Equation 6, as well as the mundane Equation 7, both tell us that the investment term τ vanishes as NPV approaches unity, and tends to infinity as the interest rate $-\iota$ tends to zero.

Thus SFEcon interprets financial collapse as instability in the savings loop insofar as the investment term τ is trying to be both zero and infinity at the same moment. The only corresponding physical reality would be sudden evacuation of the savings loop such that the void within it might move with infinite speed.

Model 1:

Model 0, having achieved sufficient development to falsify the Austrian doctrine of economic non-computability by direct counter-example, SFEcon did not elaborate further upon its elementary model of financial intermediation.

However, with the financial industry’s share of GDP approaching 20%, it was clear that empirically useful SFEcon models would require more realistic portraits of intermediation. Model 1 was the first in a succession of efforts toward this goal.

Model 1 demonstrated stable economic adjustment in which competing intermediaries consumed physical commodities that were demanded with somewhat familiar references to commodity prices and the shapes of intermediaries' utility tradeoffs. While familiar, these references could not be one-to-one:

1. The 'product' of financial intermediation is essentially time itself, i.e., the accurate hypothesization of value for a prospective asset that is only to be realized at some point in the future;

2. Thus an intermediary does not create a product with the familiar aspects of mensurable physical dimensions and a calculable marginal cost of production;

3. But an intermediary's consumption of assets must nonetheless have its proper effect on assets' prices, which means its 'production parameters' must describe indifference in terms of the same geometric form as the generic industrial sectors.

These matters were resolved by rethinking our hyperbolic descriptions of productive in difference for the case in which there is no axis for an output, hence no output variable to be optimized, and no way (within the principles of marginalism) to optimize rates of expenditure for asset replenishment.

Having arrived at satisfactory resolutions of these matters, we saw an opportunity to give place within our input/output structure to other sectors that are also beyond the conceptual boundary of marginalist causality. These included government sectors, as well as sectors for dependent populations whose output of time is input to nothing, and who therefore command no wage.

Upon concluding that the expenditure rates for dependent households and governments could be safely left to exogenous specification, we proceeded to elaborate a more advanced theory of causality for financial intermediation.

An Intermediary's Operating Margin:

Our new theory of intermediation began with a consolidation of Equation 5 in terms of a variable ν . . .

$$\nu = \left[\frac{1 - 1/\text{NPV}}{\text{T}} \right] \tag{8}$$

. . . which allows us to restate passive income ε in more compact form:

$$\varepsilon = \nu \cdot \gamma \tag{9}$$

Now if our intermediary is to operate at a profit, his harvest of dividends from his industrial clients $-\iota\kappa$ must be greater than the passive income $\nu\gamma$ he awards to his household clients. Expressed as a percentage, his margin, MAR, would be this fraction, times 100:

$$\text{MAR} = \frac{-\iota\kappa}{\nu\gamma} - 1 \tag{10}$$

Model 1's elementary model of financial intermediation can accommodate this notion of the intermediary's margin through a modification of Equation 2:

$$\text{NPV} = \frac{1}{1 + \dfrac{\iota \text{T} \kappa}{\gamma \cdot (1 + \text{MAR})}} \tag{11}$$

Here we see NPV seem enlarged by artificially making saving γ greater than accumulations of what household sectors have earned (including passive income) less what they have spent. The income from this enlargement would accrue to the intermediary through a lengthening the investment term T.

Experiments on Model 1 show that exogenous specification of MAR produces stable, efficient dynamics. More theoretically pleasing experiments that equate MAR with the interest rate ($-\iota =$ the industrial sectors' composite internal rate of return) are also dynamically stable and efficient.

Model 2:

Model 2 departed from Model 0 in a wholly different direction from that of Model 1. The problem addressed here had to do with Model 0's inadequate portrait of international finance.

It is to be observed that economic stasis tends to more or less equate a nation's trade balance with its international financial obligations. But Model 0, representing the purest anarcho-capitalism, always operates to zero-out trade imbalances – even while arriving at a global stasis characterized by robust export/import profiles in all economies.

Model 2 continued Model 0's practice of allowing only one, financially transparent, intermediary per economy. It added to our portrait of financial intermediation by modeling an international intermediary to keep the accounts arising from trade imbalances.

Model 2 showed that a national economy's international trade praxis could be specified such that stasis would arrive with trade imbalances permanently offset by static imbalances in international finance. It also showed that other specifications of trade praxis could result in stasis with a global zeroing-out of both trade imbalances and international financial accounts.

Other experiments with Model 2 exhibited stable and efficient equilibria arriving under a regime of a single universal interest rate and investment term.

Model 3:

Model 3 was undertaken to synthesize the robust intermediary of Model 1 with the international financial scope of Model 2. Its development eventuated in a much more satisfying treatment of the interest rate than our earlier formulation of $-\iota =$ the industrial sectors' composite internal rate of return.

Our latest thinking on the interest rate calculation has the financial intermediaries' margins MAR determined by pressures on their surrounding financial flows. It required two lines of mathematical reasoning based on the margin's definition in Equation 10. The first began by replacing MAR in Equation 11 with its definition from Equation 10 . . .

$$\text{NPV} = \frac{1}{1 + \frac{\iota \text{T} \kappa}{\gamma \cdot \left(-\frac{\iota \kappa}{\nu \gamma}\right)}} \qquad (12$$

. . . which reduces to a much more compact expression for net present value NPV:

$$\text{NPV} = \frac{1}{1 - \nu \text{T}} \qquad (13$$

Eliminating NPV from Equations 1 and 13 and solving for ι gives us an expression for (minus) the interest rate in terms of the intermediary's temporal parameters ν and T:

$$\iota = 1 - (1 - \nu \text{T})^{1/\text{T}} \qquad (14$$

A second line of development from Equation 10 begins with taking the derivative of the margin's definition with respect to time:

$$\frac{\partial \text{MAR}}{\partial t} = \frac{-\iota}{\nu}\left[\frac{\gamma(\partial\kappa/\partial t) - \kappa(\partial\gamma/\partial t)}{\gamma^2}\right] \qquad (15$$

By this model, margins cease to change when $\partial\kappa/\partial t$ and $\partial\gamma/\partial t$ are both zero.

The steady states for investment κ and saving γ come into being when an intermediary's contra-accounts are balanced. This occurs when κ equals the exact negative sum of all his industrial clients' investment accounts, and γ equals the exact negative sum of all his household clients' savings accounts.

The term $\partial\kappa/\partial t$ in Equation 15 is defined as $-\iota$ times the difference in magnitude between κ and the sum of industrial sectors' capital accounts; and the $\partial\gamma/\partial t$ term is defined as ν times the difference in magnitude between γ and the sum of household sectors' savings accounts.

With MAR established, along with κ and γ, as state variables within the SFEcon emulator's Euler methodology, the financial parameters, T, NPV, ν, and ι have become determined through a complicated simultaneous system: T and NPV are continuously given by a Newton-Raphson operation on Equations 1 and 11; with ν given by Equation 8; and ι emerging, at last, from Equation 14.

The observed stability of these calculations might be attributed to financial parameters' being fully implicated in the state variable MAR, – which, upon being integrated, resolves any circular references.

Stasis:

Model 3's desktop prototype features three intermediaries in each of its three national economies. The first is an investment bank harvesting proceeds from the industrial sectors, which it transmits to the households proportion to the savings they have entrusted to the bank. The second is a retail bank that harvests interest on consumer debt, mortgages, etc., which it transmits directly to its shareholders. Our third intermediary processes transactions for the two local government sectors in our prototype.

Model 3's two domestic government sectors are constrained to operate as dynamically null actors. They tax fixed amounts from the other sectors, which is exactly what they spend for the assets they consume, plus what they award to Model 3's 'dependent' household sector (which, once again) produces no economic output and therefore commands no wage).

The only purpose served by representatives of government transactions is to establish that our algorithm can accommodate fixed exogeneous transfers, even while the parties involved with those transactions can continuously and properly respond to changes in the prices of what they consume.

When Model 3 comes into stasis, both active (non-governmental) intermediaries settle down with the same financial parameters, and each sector exhibits marginalist criteria for optimality.